Journal of Nuclear Materials 313–316 (2003) 1094–1097
www.elsevier.com/locate/jnucmat

A method of kinetic description of the near-wall plasma and non-local effects

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Abstract

As is known, ion kinetics play an essential part in the formation of a near-wall plasma layer. The scatter processes with ions involved (ionization, recombination, charge exchange, etc.) influence both the value of the near-wall potential jump (and, accordingly, the value of the electric field on the surface) and the unipolar arc formation. The applicability of the Bohm criterion which is widely used in many works, also depends on the processes of ion scattering. In this paper a new method of the ion kinetics in a magnetic field is proposed which can been applied to the problems mentioned above.

Keywords: Ion kinetics; Highly inhomogeneous plasma; Non-local effects

1. Introduction

Kinetic phenomena near the boundary of the plasma and the material surface are known to be of great importance. In particular, a molecular and charge plasma composition, an 'active particle' concentration (in technological processes of microelectronics), an electric field near surface, conditions of the Bohm criterion applicability [1] and etc. are governed by these phenomena. Solution of appropriate kinetic problems is complicated by the fact that a wall plasma is usually highly inhomogeneous, i.e. particle free paths are comparable or exceed a typical dimension of spatial inhomogenity. So conventional methods used for a kinetic description (see, e.g. [2]) prove to be incorrect. A problem of the kinetic description of a highly inhomogeneous plasma was likely to arise, first, in connection with the development of a theory of arc modes of thermionic converters [3,4] and the description of non-linear modes of stratification states of a positive column of a glow discharge.

The problem of developing a theory of a highly inhomogeneous plasma in the frame of a kinetic approach has been considered, first, in a work where an appropriate method of description of electron kinetics was suggested and solutions to a number of kinetic problems were presented [3]. It was found that in a highly inhomogeneous plasma a function of the electron distribution is non-local. In another work a strict solution of kinetic equation was found for electrons in the Lorentz approximation [5]. The solution obtained in the work [3] found their further development with reference to a nonlinear theory of a stratified positive column [6,7], a theory of arc discharges of thermionic converters and a number of other areas (plasma chemistry and microelectronics technology).

A solution of the kinetic problem for ions is complicated due to the absence of a small parameter similar to the mass ratio in the case of electrons. In our work [8] a solution of the problem has been found for conditions in which the processes of charge exchange of ions and atoms are predominant. The ionization and recombination processes were also taken into account. On the basis of the obtained expression for the function of the ion distribution in velocity, which, as is in the case of electrons, was non-local, a physical–mathematical model of the wall plasma was developed and a proper numerical code was prepared. The calculations performed showed, that the employment of simplified models which became

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0022-3115/03/$ - see front matter
PII: S0022-3115(02)01544-1

commonly used could lead to incorrect results. For instance, the Bohm criterion, the value of the potential and the electric field near the surface, the ion flux velocity in the wall direction are substantially different from the results obtained with the simplified models. A physical interpretation of the results obtained is presented in work [9] (for the plasma in the magnetic field appropriate data can be found in [10]).The solution technique for the problem of the kinetic description of ions in a highly inhomogeneous plasma, suggested in work [8], did not include the magnetic field. Below the data on the ion kinetics [8] are generalized for the case of the plasma in the magnetic field.

2. Ion kinetics study

2.1. On a description of ion kinetics in a wall plasma with no magnetic field

Consider the kinetics of ions in the gas of the same type. We neglect the processes of ion elastic collisions and take into account ionization and recombination which produce a significant effect on the particle balance. Assuming that the charge exchange frequency does not depend on the velocity, the kinetic equation for ions is simplified: (It is assumed the plasma parameters depend on the x-coordinate only, where the x-axis is perpendicular to the wall.)

$$\frac{\mathrm{d}f}{\mathrm{d}t} = v_x \frac{\partial f}{\partial x} + \frac{eE_x}{m_\mathrm{i}} \frac{\partial f}{\partial v_x} = \frac{n_\mathrm{i} f_0}{\tau_1} - \frac{f}{\tau_2}. \tag{1}$$

Here, n_i is the ion concentration, $f_0 = f_0(v)$ is the normalized function of the neutral particle distribution, E_x is the electric field in plasma and $\tau_1^{-1} = \tau^{-1} + (n_\mathrm{e}/n_\mathrm{i})\tau_\mathrm{e}^{-1}$, $\tau_2^{-1} = \tau^{-1} + \beta n_\mathrm{e}^2$ where τ is the characteristic time of charge exchange, τ_e is the characteristic time of ionization by electron impact and β is the constant of recombination. Solution (1) was found in [8] from the following considerations. As is known, in accordance with the Liouville theorem in a collisionless plasma the function of particle distributions in velocity depends only on motion integrals. In the case of collisions it is necessary to allow for the dependence of the distribution function on the spatial x-coordinate, i.e. the distribution function can be sought in the form of $f = f(\varepsilon_x, x)$, where $\varepsilon_x = m_\mathrm{i} v_x^2/2 + e\varphi(x)$.

After introduction the new variables Eq. (1) is transformed to the form:

$$v_{x'} \frac{\partial f}{\partial x'} = \frac{n_\mathrm{i} f_0}{\tau_1} - \frac{f}{\tau_2}, \tag{2}$$

where $x' = x$ and $v_{x'} = (2(\varepsilon - e\varphi)/m_\mathrm{i})^{1/2}$.

It is convenient to represent the distribution function sought as two summands conforming to the particle movement in the x-axis direction (index +) or in the opposite direction (−):

$$f = f^+(v_x > 0, x) + f^-(v_x < 0, x).$$

Assuming that the ions fully recombine on the surface which is taken as the origin, we have for particles with $v_x > 0$:

$$f^+(v_x > 0, 0) = 0. \tag{3}$$

Far from the surface the neutral distribution function will be considered Maxwellian,

$$f(v_x, \infty) = n_\mathrm{i} f_0(v_x)\tau_2/\tau_1. \tag{4}$$

With boundary conditions from (3) and (4) the solution of Eq. (3) has the form:

$$f^+(v_x > 0) = \int_0^x \mathrm{d}\xi \frac{n_\mathrm{i} f_0'}{\tau_1 v_\xi} \exp\left(- \int_\xi^x \frac{\mathrm{d}\xi'}{\tau_2 v_{\xi'}} \right). \tag{5}$$

The following designations are used above:

$$v_\xi = \sqrt{v_x^2 + 2e[\varphi(x) - \varphi(\xi)]/m_\mathrm{i}}, \tag{6}$$

$$v_{\xi'} = \sqrt{v_x^2 + 2e[\varphi(x) - \varphi(\xi')]/m_\mathrm{i}}, \tag{7}$$

$$f_0' = \left(\frac{m_\mathrm{i}}{2\pi T}\right)^{1/2} \exp\left[- \frac{m_\mathrm{i} v_\xi^2}{2T} \right], \tag{8}$$

where T is the temperature of neutral particles.

The ion born at point ξ with velocity v_ξ, at no scattering, will have velocity v_x at point x. Above, $v_{\xi'}$ is the velocity of this ion at the intermediate point ξ'. The expression for f^- depends on the absolute value of velocity. If $|v_x| > (-2e\varphi(x)/m_\mathrm{i})^{1/2}$,

$$f^-(v_x < 0) = \int_x^\infty \mathrm{d}\xi \frac{n_\mathrm{i} f_0'}{\tau_1 v_\xi} \exp\left(- \int_x^\xi \frac{\mathrm{d}\xi'}{\tau_2 v_{\xi'}} \right). \tag{9}$$

But if $|v_x| < (-2e\varphi(x)/m_\mathrm{i})^{1/2}$, we have the 'turning point' $\zeta > x$, so $m_\mathrm{i} v_x^2/2 + e\varphi(x) = e\varphi(\zeta)$ and the particles which have velocity v_x at point x are divided into two groups. The first ones were born between x and ζ at a velocity $v_\xi < 0$, and the particles of second group were born between the wall ($x = 0$) and ζ at velocity $v_\xi > 0$ and then they changed the velocity direction when reached the turning point. Both particle groups give the first and the second summand in the expression for f^-, accordingly:

$$f^-(v_x < 0, x) = \int_x^\varsigma \mathrm{d}\xi \frac{n_\mathrm{i} f_0'}{\tau_1 v_\xi} \exp\left(- \int_x^\xi \frac{\mathrm{d}\xi'}{\tau_2 v_{\xi'}} \right) + \int_0^\varsigma \mathrm{d}\xi \frac{n_\mathrm{i} f_0'}{\tau_1 v_\xi} \times \exp\left(- 2\int_x^\varsigma \frac{\mathrm{d}\xi'}{\tau_2 v_{\xi'}} - \int_\xi^x \frac{\mathrm{d}\xi'}{\tau_2 v_{\xi'}} \right). \tag{10}$$

If the charge exchange frequency is small and $\tau = \infty$ we have, according to (5), (9) and (10), a modified Maxwellian distribution of type (8) for $v_x < 0$ (for the particle flying to the surface); for $v_x > 0$ we have $f^+ = 0$, i.e. the particles reach the surface after having crossed large distances and recombine. In the opposite case where the charge exchange frequency is high and $\tau = 0$ there occurs a modified Maxwellian distribution for $v_x > 0$ and $v_x < 0$.

Expressions (5), (9) and (10) were used in work [8] for a numerical study of the wall plasma. These expressions also can be employed for processing probe data. Recall that the function of ion distribution in velocity (FID) represented as expressions (5), (9) and (10) remains valid at an arbitrary frequency of scattering (ion path length).

2.2. Ion kinetics in magnetic field

The approach to the investigation of all of the wall plasma characteristics suggested in work [8] can be successfully used in the case of the magnetic field. A strict solution of the problem requires that the kinetic equation for ions be solved together with the Poisson equation for potential. The Boltzmann integral–differential equation, in the very general case, is reduced to a pure integral equation for FID using the method of characteristics. In some cases one can write out an entire system of first integrals of collisionless motion of a single ion. If the equation of ion motion can be integrated analytically, the characteristics are written down explicitly as is the case with the integral equation for FID. The perturbation theory allows the cases close to those integrated to be studied analytically. To illustrate this concept, a case was considered where the plasma is in a magnetic field which is directed strictly along the wall. In other respects the problem statement was made in Section 2.1. The kinetic equation takes on the form:

$$\frac{\mathrm{d}f}{\mathrm{d}t} = v_x\frac{\partial f}{\partial x} + \frac{eE_x}{m_\mathrm{i}}\frac{\partial f}{\partial v_x} + \omega_\mathrm{H} v_y \frac{\partial f}{\partial v_x} - \omega_\mathrm{H} v_x \frac{\partial f}{\partial v_y} = \frac{n_\mathrm{i} f_0}{\tau_1} - \frac{f}{\tau_2}. \tag{11}$$

Here ω_H is the ion cyclotron frequency. The Boltzmann equation does not depend on the z-coordinate. It is quite easy to show that there are two integrals of motion: $I_1 = v_x^2/2 + v_y^2/2 + e\varphi(x)/m_\mathrm{i}$, $I_2 = v_y + \omega_\mathrm{H} x$.

For simplicity, we take $T = 0$, i.e. the ions are born as a result of charge exchange or ionization at a zero velocity. Far from the wall the motion along the x-coordinate is periodic, the ion moves between two points: ξ and ζ ($\xi > \zeta$). At point ξ the ion was born with a zero velocity and ζ is the turning point, where $v_x = 0$ and $v_y \neq 0$. ξ and ζ can be found from the equations:

$$v_x^2/2 + \omega_\mathrm{H}^2(\xi - x)^2/2 + e[\varphi(x) - \varphi(\xi)]/m_\mathrm{i} = 0,$$

$$\omega_\mathrm{H}^2(\xi - \zeta)^2/2 + e[\varphi(\zeta) - \varphi(\xi)]/m_\mathrm{i} = 0.$$

Let us designate:

$$s = \int_x^\xi \frac{\mathrm{d}\xi'}{\tau_2\sqrt{2[\varphi(\xi) - \varphi(\xi')]e/m_\mathrm{i} - \omega_\mathrm{H}^2(\xi - \xi')^2}},$$

$$S = 2\int_\zeta^\xi \frac{\mathrm{d}\xi'}{\tau_2\sqrt{2[\varphi(\xi) - \varphi(\xi')]e/m_\mathrm{i} - \omega_\mathrm{H}^2(\xi - \xi')^2}},$$

where s denotes a relative number of ions passed from ξ to x moving less than a half period and S denotes the relative number of ions returned to ξ having made one turn. By integrating the Boltzmann equation we shall have an expression for the FID:

$$f^+(v_x > 0, v_y, x) = -\frac{n_\mathrm{i}(\xi)\delta(v_y + \omega_\mathrm{H}(x - \xi))}{\tau_1 eE(\xi)/m_\mathrm{i}}\frac{\exp(-S + s)}{1 - \exp(-S)}, \tag{12}$$

$$f^-(v_x < 0, v_y, x) = -\frac{n_\mathrm{i}(\xi)\delta(v_y + \omega_\mathrm{H}(x - \xi))}{\tau_1 eE(\xi)/m_\mathrm{i}}\frac{\exp(-s)}{1 - \exp(-S)}. \tag{13}$$

Here, δ is the symbol for the δ-function. If the movement along the x-coordinate is aperiodic (e.g. we see the FID near the wall which absorbs all the ions on it), the formulas will change:

$$f^+(v_x > 0, v_y, x) = 0, \tag{14}$$

$$f^-(v_x < 0, v_y, x) = -\frac{n_\mathrm{i}(\xi)\delta(v_y + \omega_\mathrm{H}(x - \xi))}{\tau_1 eE(\xi)/m_\mathrm{i}}\exp(-s). \tag{15}$$

3. Conclusions

The proposed method of analytical investigation into the ion kinetics permits a correct physical–mathematical model of a highly inhomogeneous near-wall plasma and also a diverter plasma to be formulated. On the basis of this model a numerical code could be developed that will help to conduct a more strict investigation of the wall plasma than with a conventional approximation.

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